Local Energy Markets*

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Abstract

In current power markets, the bulk of electricity is sold wholesale and transported to consumers via long-distance transmission lines. Recently, decentralized local energy markets have evolved, often as isolated networks based on solar generation. We analyze strategic pricing, investment, and welfare in local energy markets. We show that local energy markets yield competitive equilibrium prices and provide efficient investment incentives. Yet, we find that strategic behavior leads to allocative inefficiency. We propose a clearing mechanism that induces truth-telling behavior and restores first-best welfare.

Keywords: Market Design, Networks, Peer-to-Peer Markets, Electricity.
JEL-Classification: D16, D26, D47, L94.

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I. Introduction

Electricity access for all has become a major topic for international energy, climate, and regulatory policy. Especially in developing economies, small energy systems based on solar generation increasingly provide rural areas with electricity. Local peer-to-peer markets, eventually separated from large scale power grids, have however also been field-tested in established electricity systems.[1]

In this article, we provide a model for studying electricity provision in local energy markets. The model allows to analyze pricing behavior and investment incentives, and to quantify welfare implications that arise from strategic behavior of participating households. Because we find that strategic behavior can lead to allocative inefficiency, we also propose a clearing mechanism for local energy markets that alleviates strategic behavior and restores first-best welfare.

The markets that we analyze have been emerging as alternatives to costly expansion of large-scale grids (Fowlie et al., 2019). More specifically, local energy markets consist of a microgrid that can operate self-sufficiently and enable trade of locally generated energy between all connected households. While microgrids can work in stand-alone mode, they are also capable of interconnecting with other microgrids and, if available, to the greater distribution and transmission grid (Urpelainen, 2014). Due to their ability for operating stand-alone, local energy markets are however more common in rural areas with no or unreliable access to transmission grids (Perez-Arriaga et al., 2019). Abundant examples of interconnecting off-grid consumers in isolated local markets can currently be found in African and South-Asian countries, such as Bangladesh, where pilot peer-to-peer trading is emerging amongst prosumer households.[2] Because we study isolated energy markets without a connection to the

[1]Established electricity systems consist of complex vertical and sequential market arrangements. Wilson (2002) presents an overview on typical electricity market architecture.

[2]For the Bangladesh market, see https://me-solshare.com.

transmission grid, our model is mostly tailored to local markets in rural areas that operate in stand-alone mode.

In the environment that we study, households can decide to participate in a local market for electricity, where all participating households are connected to an isolated local microgrid. Households participate by demanding electricity from the local microgrid and by investing in generating plants, here solar plants, that add supply to the local network. Households that have invested in generation capacity can be net-selling or net-buying from the market, depending on whether their generation covers more or less than their consumption. Given aggregate household demand and aggregate solar supply, the market constitutes an equilibrium price for electricity. We model market clearing using a demand function approach. Having characterized pricing and investment equilibria in local energy markets, we subsequently analyze welfare implications and propose a clearing mechanism that induces truth-telling behavior and guarantees first-best welfare.

Our results on pricing and investment outcomes in local markets are substantially different from traditional energy markets, composed of top-down generation, transmission, and retail supply chains. In particular, we find that local energy markets yield competitive market prices. Importantly, this result holds even for a small number of market participants (i.e., households or prosumers) and despite strategic pricing behavior by all participating households. The market price remains competitive because *both* net-buying and net-selling households exercise strategic pricing, and the impact of their strategies on equilibrium prices cancels out. In essence, households engage in demand reduction in a similar fashion as in Ausubel et al. (2014). Where households have invested in production plants and are net-sellers to the market, they however inflate their demand to push up market prices. In equilibrium, demand reduction and demand inflation cancel out, ensuring competitive prices.[3]

[3]Intuitively, equilibrium prices are competitive, because the incentives of net-buying and net-selling households are the same. To see this, consider a household i that buys energy from the market. Its equilibrium demand reduction has to be optimal against the competing demand functions from all other households

Furthermore, we show that competitive prices provide efficient investment incentives.

Although market prices are competitive and investment is efficient, we find that strategic pricing behavior can cause allocative inefficiency. Allocation is not optimal as, intuitively, net-buyers aim to decrease prices by announcing lower demand, while net-selling households aim to increase prices by announcing relatively higher demand. In equilibrium, prices are competitive but households' consumption levels still follow their strategically announced demand profiles. As a result, allocative inefficiency arises and reduces welfare.

We show that altering payment rules can increase allocative efficiency and restore first-best welfare. Specifically, we propose network tariffs that can be organized by a microgrid operator and which complement the uniform energy prices. These payments make it optimal to announce truth-telling demand and thereby guarantee first-best welfare. The payments are composed of a fixed fee paid by the households for participating in the market, similar to a one-off network connection charge, and a payment that is a function of whether households are net-sellers or net-buyers. Importantly, the payments can be organized such that the microgrid operator breaks-even when organizing the market. Our mechanism hence is feasible and budget balanced, and only consists of energy prices and network tariffs.

The main idea of this mechanism draws from the regulation of utilities as proposed by Loeb and Magat (1979). In our application, the one-off network charges paid by households to the microgrid operator correspond to the franchise bidding stage in Loeb and Magat (1979), i.e., they allow households to participate in the local market. The variable incentive payments that the microgrid operator then pays to the households correspond to the subsidy stage in Loeb and Magat (1979). One important difference is that energy demand, and thus the incentive payments organized by the microgrid operator, are uncertain, and hence the microgrid operator breaks-even *in expectation*.

$j \neq i$. If household i instead sells energy to the market, its optimal supply likewise depends on the demand functions of all households $j \neq i$. Hence, net-buying and net-selling households maximize against the same set of competing demand functions.

Finally, we present several corollary results relevant to the organization of local energy markets. For instance, our results show that welfare increases the more households participate. This finding highlights the relevance of interoperability of local markets, that often rely on different technology standards. While some microgrid systems in developing economies use DC networks, other systems rely on AC networks, especially where markets are larger (US AID, 2017).[4] Our results hence show that investing in interoperability can raise welfare and should be fostered in local energy markets.

Our findings contribute to several strands of research. First, our findings contribute to the large literature on electricity market architecture. Beginning with the deregulation of this sector, the literature has paid significant attention to the different levels of the supply chain, i.e., wholesale markets (Newbery, 1998; Wolfram, 1999; Borenstein et al., 2000; Fabra et al., 2006; Bushnell et al., 2008; Reguant, 2014; Schwenen, 2015), retail market design and consumer behavior (Joskow and Tirole, 2006; Allcott, 2011; Allcott and Rogers, 2014; Giulietti et al., 2014; Poletti and Wright, 2020), and network regulation (Joskow, 2008; Tangerås, 2012). More recently, the literature started investigating the design of electricity markets with high shares of low-carbon generation (Holmberg and Wolak, 2018; Tangerås and Mauritzen, 2018) and the use of distributed generation (Brown and Sappington, 2017). We add to this literature by providing a model on the efficiency of local energy markets, where strategic households simultaneously act as producers, consumers, and traders, and where we abstract from the canonical producer to consumer, wholesale to retail market architecture.

The operation of local energy markets has also been covered by interdisciplinary approaches at the intersection with the engineering and operations research literature. This

[4]Currently applied DC systems have relatively higher losses and often are not rolled out for more than 200 meters. In addition, most available appliances connected to local energy systems are running on AC (US AID, 2017). AC can be converted to DC using rectifiers, while DC can be converted to AC by using inverters. We do not model competition between standards and focus on efficiency and market design within one network. For conditions under which common network standards can generally be achieved see, e.g., Farrell and Saloner (1988) and Baake and Boom (2001).

stream of research explicitly focuses on the underlying physical attributes of local energy markets, such as network constraints, consumption characteristics, power flows, network stability, and the IT infrastructure. Dumitrescu et al. (2020) describe the underlying applications, usage characteristics, and consumption patterns in local energy markets. Pinto et al. (2021) discuss relevant IT tools and different concepts for trading platforms, emphasizing the importance of the required physical and software infrastructure. Khorasany et al. (2020) and Tushar et al. (2020) study different peer-to-peer algorithms and market clearing techniques and stress the need to efficiently match supply and demand for households with different demand patterns. While our model does not share the same granularity as the more technical literature, we adopt as many of the techno-economic aspects as possible. In particular, our model and proposed clearing mechanism for inducing truth-full behavior contribute to the stream of this literature that explores clearing algorithms in local energy markets.

We also relate to the literature on electricity access and the benefits of electrification. Dinkelman (2011) finds positive effects on employment of a large grid connection plan in South Africa. Lee et al. (2020) provide experimental evidence on electrification in rural Kenya, and identify scale economies of connecting households to the power grid. Reporting evidence from India, Aklin et al. (2016) find that only a few hours of additional electricity supply increases household satisfaction substantially. Comello et al. (2017) find that rural electrification lags behind policy goals and identify the threat of grid extension as a barrier for further investment. We add to this growing literature on electrification with a theoretical study. To our knowledge, no analytical model has so far been developed to understand efficiency and regulatory requirements for the electrification through local energy markets.

Last, our model adds to the literature on markets that follow double auction formats.[5] Beginning with Wilson (1979), Smith et al. (1982), and Klemperer and Meyer (1989), this

[5] See Friedman and Rust (eds.) (1993) for an overview.

literature has studied market interaction via demand and supply functions. The supply function framework has been widely used to study wholesale power markets (Green and Newbery, 1992; Baldick et al., 2004; Hortacsu and Puller, 2008), while demand function equilibria have been used to model strategies of traders with different information sets (Kyle, 1989) and in sequential double auctions (Du and Zhu, 2017). Our model draws from the demand function equilibrium approach, that we amend to study equilibrium pricing in peer-to-peer markets for energy.

The remainder is organized as follows. Section two discusses the institutional environment and outlines a model for local energy markets. In section three, we study equilibrium pricing and investment. Section four presents a clearing mechanism that guarantees truth-telling behavior and first-best welfare. Section five concludes and discusses policy implications.

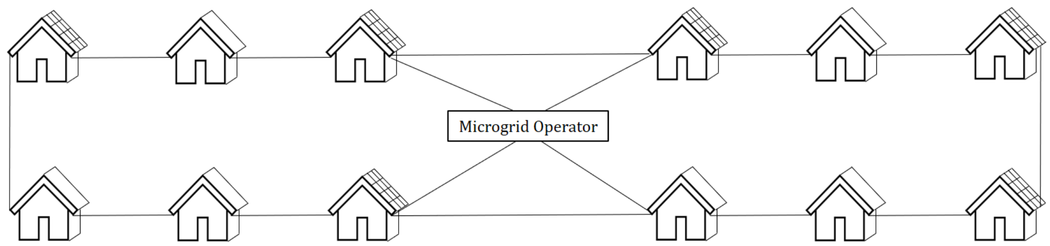
II. A model of local energy markets

Market participants and technology. Figure 1 shows the stylized market environment. We study a local energy market where n households are connected to a microgrid, and have no access to the larger transmission network. Each household has demand for electricity and can invest in generation assets (here solar rooftop plants) to generate and consume electricity. In case of excess electricity, each household can sell to all $n-1$ neighbors via the common microgrid. Vice versa, when own generation capabilities are exhausted, each household can buy electricity from its neighbors. Depending on the amount of installed solar plants relative to aggregate demand, local energy trade establishes an equilibrium price for electricity.[6]

We analyze energy trading that is organized through the matching of supply and demand from utility-maximizing households. We do not specify the IT infrastructure in which trading

[6] In addition, storage units can operate in local energy markets, arbitrage energy prices, and increase grid stability. We exclude storage units from the analysis. Andrés-Cerezo and Fabra (2022) show that the efficiency of energy storage depends on market structure and ownership. Including independently owned storage units with no capacity constraints and linear bid functions does not change the results in our model.

Figure 1: *Schematic representation of local energy market with rooftop solar plants.*



takes place, but assume that the installed infrastructure allows households to communicate their demand for energy to a central market clearing entity, i.e., a microgrid operator. We assume that the microgrid operator is not vertically integrated and has no incentives to behave strategically.[7] We do not consider transactive energy and other technical issues related to network stability, such as backup reserve or balancing markets. Instead, we focus on the role of the microgrid operator in organizing the market and allow the operator to collect network tariffs for this sake.

Market clearing and prices. As introduced above, households can face energy prices and network payments. We consider a market where households announce their demand for electricity, while supply is determined by all generating resources that are connected to the microgrid. Hence, all production units are pooled, produce at full output, and cannot be used strategically. Aggregate demand and supply determine the price for electricity, equilibrium consumption, and net-selling (buying) positions of each household. We model market clearing as a one shot game, in which households submit their demand functions, and abstract from whether households specify demand for a course of an hour, day, or any specified peak or off-peak period.[8] We assume that all households are connected to the

[7]Microgrid operators have a similar role as independent system operators that manage high-voltage transmission grids, and can also be the owners of the local grid. Wang and Huang (2015) provide an analysis of microgrids with vertically integrated operators that also own generation assets.

[8]Local energy markets often feature both network tariffs and variable energy prices. For instance, time-variant energy prices may account for varying demand during day and night. For a detailed description of tariff mechanisms see US AID (2017).

microgrid, and that households who do not invest in solar plants hence must rely on buying energy from their neighbors.

In addition to the energy price, we study network tariffs in the local market. We consider network payments that can be organized by the microgrid operator and which complement the energy price. We model network tariffs that are composed of a fixed fee paid by the households for participating in the market, similar to a one-off network connection charge, and a payment that is a function of how much households consume, similar to a volumetric network tariff in traditional distribution systems (Azarova et al., 2018). As we show, network tariffs can be organized such that they incentivize truth-telling behavior, while allowing the microgrid operator to break-even when organizing the market. Our market mechanism hence is feasible and budget balanced, and only consists of energy prices and network payments.

Preferences. The utility of each household i is denoted as $U_i(x_i, \varepsilon_i)$ and is concave in x_i. Utility depends on the realization of an idiosyncratic error term, ε_i, known only to household i. Formally, we assume

$$U_i(x_i, \varepsilon_i) = (\theta_i + \varepsilon_i)x_i - \frac{1}{2}x_i^2. \tag{1}$$

The realization of the demand shock is known to household i prior to participating in power trading, and we assume that shocks are independent. We hence consider an independent private value setting throughout the power trading stage.[9] Because the utility of household i is private information, the demand of its neighboring households is random and hence the equilibrium power price is uncertain.

[9] As discussed further below, our model takes into account that households, however, do not yet know their realized demand shock at the investment stage, this is, when they decide on their investment in generation assets. Put differently, households maximize ex-ante utility at the investment stage and interim utility at the pricing stage.

III. Equilibrium pricing

Households simultaneously decide on their consumption schedule $X_i(p, \varepsilon_i)$ that specifies their demand from the local grid at each price p. In equilibrium, each household consumes $X_i(p^*, \varepsilon_i)$ where p^* is the equilibrium price that equates supply and demand. Formally, the equilibrium price is given by

$$p^* : \sum_i^n X_i(p, \varepsilon_i) = \sum_i^n q_i, \tag{2}$$

with q_i being the installed generation capacity of household i and the sum is taken over all households $i = 1, ..., n$. A household's profit from trading electricity becomes $p(q_i - X_i(p, \varepsilon_i))$. Consequently, the quasi-linear utility from consumption and trade is

$$U_i\left(X_i(p, \varepsilon_i), \varepsilon_i\right) + p(q_i - X_i(p, \varepsilon_i)). \tag{3}$$

Suppose that the idiosyncratic consumption shock ε_i is drawn from a distribution F prior to submitting demand schedules. Let the support of ε_i be equal for all households, finite with $\varepsilon_i \in [-\varepsilon_o, \varepsilon_o]$, and symmetric around $\mathbb{E}[\varepsilon_i] = 0$. Because the type of each neighboring household (i.e., their realized demand) and therefore the clearing price is unknown prior to announcing demand, strategies must be Bayesian-Nash optimal. Households must maximize expected utility and, before deciding on $X_i(p, \varepsilon_i)$, form an expectation on aggregate demand and the equilibrium price.

To capture the uncertainty in price, conditional on household i's demand function, we draw from the auction literature (Wilson, 1979; Hortacsu and Puller, 2008) and use the market clearing condition in equation (2) to map randomness from demand to price. Specifically, the distribution function of the equilibrium electricity price p^*, given household i's demand

X_i at price p, becomes

$$\text{(4)} \qquad \begin{aligned} H_i(p, X_i(p, \varepsilon_i)) &= Pr\left(p^* \leq p \mid X_i\right) \\ &= Pr\left(\sum_{j \neq i}^{n} X_j(p, \varepsilon_j) + X_i \leq \sum_{i}^{n} q_i \;\middle|\; X_i\right), \end{aligned}$$

where the sum is taken over all n households except household i. The distribution function H_i states the probability that $p^* \leq p$, this is, the probability that supply is larger than demand at this price. The support of H_i on $[\underline{p}, \bar{p}]$ depends on the support of all idiosyncratic demand shocks.

Using this probability measure, the expected utility of household i can be written as

$$\text{(5)} \qquad EU_i = \int_{\underline{p}}^{\overline{p}} \left[U_i\left(X_i(p, \varepsilon_i), \varepsilon_i\right) + p(q_i - X_i(p, \varepsilon_i))\right] dH_i(p, X_i(p, \varepsilon_i)).$$

Households maximize expected utility by specifying optimal demand over the range of possible prices. In optimum, a household is indifferent between shifting demand from one possible price level to another. Formally, as shown in Appendix A.I., optimality is given by the Euler-Lagrange first order condition, which after rearranging yields:

$$\text{(6)} \qquad \frac{\partial U_i(X_i(p, \varepsilon_i), \varepsilon_i)}{\partial X_i} - p = \left(q_i - X_i(p, \varepsilon_i)\right) \frac{H_{X_i}(p, X_i(p, \varepsilon_i))}{H_p(p, X_i(p, \varepsilon_i))},$$

where H_{X_i} and H_p are the derivatives of H_i with respect to X_i and p.[10] To interpret the optimality condition in equation (6), note that H_p is the probability density function of price and must be positive. In contrast, H_{X_i} must be negative, because additional demand decreases the likelihood that the price is below any given value. Consequently, $\frac{H_{X_i}}{H_p} < 0$, and equation (6) shows that in equilibrium households that are net-sellers to the local market must have marginal utility from consumption below the market price. We summarize this

[10] We implicitly assume that for all q_i and ε_i we have $X_i > 0$ and $p \geq 0$.

finding in the following Proposition.

Proposition 1. *Households that are net-sellers to the local market mark-up sales above their marginal utility of consumption. Households that are net-buyers from the local market mark-down demand below their marginal utility of consumption, i.e.,*

$$\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} < p \iff q_i > X_i(p,\varepsilon_i)$$

$$\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} > p \iff q_i < X_i(p,\varepsilon_i).$$

Proof. The result follows from equation (6). A detailed proof of the first order condition is shown in Appendix A.I.. □

The optimality condition suffices for computing the equilibrium demand strategies, given model primitives for household utility on the left hand side of equation (6). Notice that also the derivatives of H_i on the right hand side of equation (6) depend on the functional form of utility and its corresponding demand function. For this reason, equation (6) can be evaluated analytically only in few cases (Hortaçsu, 2011). We hence assume utility as in equation (1) above, which exhibits saturation and results in linear demand.[11]

Next, we first derive a household's optimal strategy, given linear demand, and further below prove that equilibria in linear strategies indeed exist. Given utility as in equation (1), household i's marginal utility is given by

$$\frac{\partial U_i(x_i,\varepsilon_i)}{\partial x_i} = \theta_i + \varepsilon_i - x_i. \tag{7}$$

[11] The focus on linear bid functions is common in the literature. Du and Zhu (2017) prove that linear demand equilibria exist when trading takes place in sequential double auctions. Foster and Viswanathan (1996) find linear equilibria when traders learn from other traders' signals. Vives (2011) analyzes linear supply function strategies where sellers' cost functions include common and private value components. Baldick et al. (2004) presents an overview of different types of supply function equilibria, including cases where sellers face capacity limits.

The parameter θ_i represents a household's maximal willingness to pay (for $\varepsilon_i = \mathbb{E}[\varepsilon_i] = 0$) and can be viewed as a parameter that specifies household size. We rank households sizes as $\theta_1 > \theta_2 > ... > \theta_n$. Given true willingness to pay of $\theta_i + \varepsilon_i - x_i$ as indicated above, each household can shade its demand and announce a linear demand function of the form

$$X_i(p, \varepsilon_i) = \alpha_i + \beta_i \varepsilon_i - \gamma_i p, \tag{8}$$

where α_i, β_i, and γ_i are choice variables for each household. Put differently, households can hide their reservation value by announcing α_i instead of θ_i and can shade their sensitivity to the error term and price by choosing β_i and γ_i.

Substituting demand as specified above in (8) into the optimality condition in (6) and using that, as shown in Appendix A.I., $\frac{H_{X_i}}{H_p} = \frac{-1}{\sum_{j \neq i}^{n} \gamma_j}$, we can rewrite a household's optimality condition to

$$\underbrace{\theta_i + \varepsilon_i - p - (\alpha_i + \beta_i \varepsilon_i - \gamma_i p)}_{\text{Difference between true and strategic demand}} + \frac{1}{\sum_{j \neq i}^{n} \gamma_j} \underbrace{(q_i - (\alpha_i + \beta_i \varepsilon_i - \gamma_i p))}_{\text{Net position}} = 0. \tag{9}$$

This rearranged optimality condition shows that the extent of strategic demand reduction (inflation) equals a household's net position, adjusted for the slope of competing demand functions $\frac{1}{\sum_{j \neq i}^{n} \gamma_j}$. Moreover, equation (9) reveals that the incentives to strategically announce higher or lower demand both depend on the slope of competing demand, and that the marginal impact of $X_i(p, \varepsilon_i) = \alpha_i + \beta_i \varepsilon_i - \gamma_i p$ on the left hand side of (9) is equal to

$$-\left(1 + \frac{1}{\sum_{j \neq i}^{n} \gamma_j}\right) \tag{10}$$

and independent of whether a household is net-buying or net-selling. Hence, the marginal incentives to decrease (increase) their quantities $X_i(p, \varepsilon_i)$ are symmetric for net-buying (net-selling) households.

Applying coefficient matching to equation (9), we find that for $n \geq 3$ households and positive trading volumes, there exists a unique strategy in linear demand functions. As shown in Appendix A.II., equilibrium demand functions $X_i^*(p, \varepsilon_i)$ are characterized by

$$\alpha_i = \frac{q_i + \theta_i \sum_{j \neq i}^{n} \gamma_j}{1 + \sum_{j \neq i}^{n} \gamma_j} \quad \text{and} \quad \beta_i = \gamma_i = \frac{\sum_{j \neq i}^{n} \gamma_j}{1 + \sum_{j \neq i}^{n} \gamma_j}. \tag{11}$$

This equilibrium is in stark contrast to truthful demand of $\alpha_i = \theta_i$ and $\beta_i = \gamma_i = 1$. As can be seen, bid shading for the reservation value, α_i, depends on the household's amount of solar plants. Moreover, the steepness of household i's demand function, γ_i, depends on the slope of other households' demand functions. This is intuitive because in equilibrium each household optimizes its demand schedule vis-à-vis the slope of its residual supply, which in turn is determined by the demand functions of its neighbors. This equilibrium feature reveals the complementarity in demand strategies: The more price-sensitive the demand of household i's neighbors becomes (the more $\sum_{j \neq i}^{n} \gamma_j$ in equation (9) increases), the less can household i impact the market price and thus has little incentives to deviate from announcing true marginal utility. As stated in the following Proposition, this complementarity results in symmetric equilibrium demand.

Lemma 1. *Households submit symmetric demand functions, conditional on their size θ_i and installed generation capacity q_i. In markets with $n \geq 3$, the unique equilibrium demand function parameters are*

(1) $\gamma_i^* = \frac{n-2}{n-1}$

(2) $\beta_i^* = \frac{n-2}{n-1}$

(3) $\alpha_i^* = \frac{q_i + \theta_i(n-2)}{n-1}$.

With $n < 3$, no trade occurs in the local market and each household consumes its own electricity.

Proof. First, we show in Appendix A.II. that trade ceases for $n < 3$. For $n \geq 3$ and given symmetry, rearrange γ_i from equation (11) to $\gamma_i = \frac{\sum_{j \neq i}^{n} \gamma_j}{1+\sum_{j \neq i}^{n} \gamma_j} = \frac{(n-1)\gamma_i}{1+(n-1)\gamma_i}$ $\forall\ i$. Solving for γ_i yields $\gamma_i^* = \frac{n-2}{n-1}$. The equilibrium parameters α_i^* and β_i^* follow immediately. Appendix A.II. further shows that asymmetric strategies cannot exist if $\gamma_i > 0$. □

So far, the strategies in Lemma 1 rely on the supposition of linear demand, as stated in equation (8).[12] Next, we prove that there exists an equilibrium in linear demand functions.

Proposition 2. *There exists an equilibrium in linear demand functions.*

Proof. Given that all households $j \neq i$ submit linear demand schedules $X_j(p, \epsilon_j)$ with any constant slope γ_j, we show that the best reply of household i is to also submit a linear demand function. We present the full proof in Appendix A.III., including a proof of the sufficiency conditions for linear equilibrium strategies. □

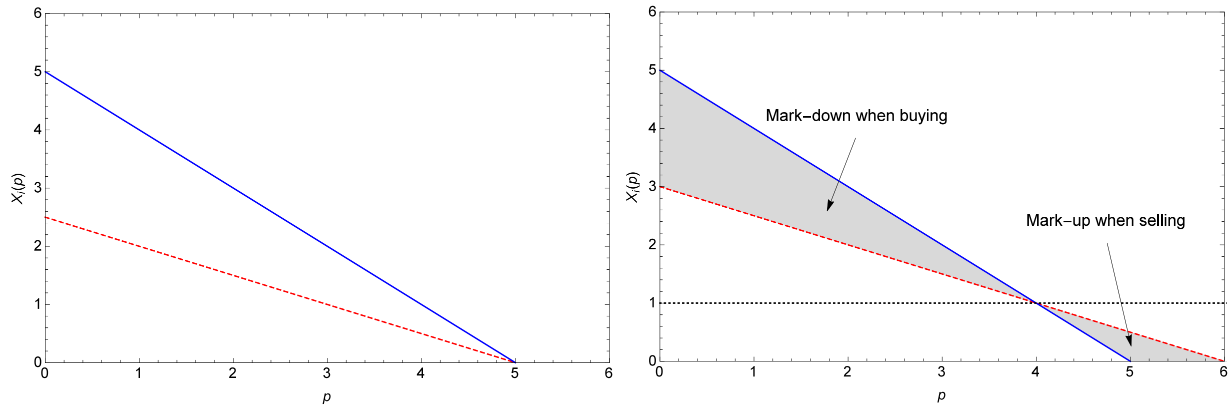
The strategy in Lemma 1 consequently belongs to the class of equilibria as derived in the above proposition. Moreover, from Lemma 1 it follows that $\lim_{n\to\infty} \alpha_i^* = \theta_i$ and $\lim_{n\to\infty} \beta_i^* = \lim_{n\to\infty} \gamma_i^* = 1$, so that strategic demand shading ceases for a large number of neighbors.

Figure 2 illustrates equilibrium demand functions for three households so that $\beta_i = \gamma_i = \frac{1}{2}$. Panel (a) depicts true demand (solid line) and strategic demand (dashed line) for a representative household with $\theta_i = 4, \varepsilon_i = 1$, and $q_i = 0$. Panel (b) shows true demand (solid line), strategic demand (dashed line), and solar capacity (dotted horizontal line) for a household with $\theta_i = 4, \varepsilon_i = 1$, and $q_i = 1$.

As can be seen in panel (a), households that do not produce electricity strategically announce lower demand and mark-down their demand at any price. This strategy is similar

[12]Note that the equilibrium strategy in Proposition 1 is also ex post optimal, i.e., after households have observed the realized demand functions of other households. This result follows because the uncertainty in demand is additive. Following arguments in Hortacsu and Puller (2008), all uncertainty, from the perspective of household i, shifts the residual supply curve but does not rotate it. The optimal demand function yields a pointwise best-response to every possible realization of the demand from competing households. Put differently, only the slope of demand, $\sum_{j \neq i}^{n} \gamma_j$, but not the realized level of demand is relevant for the best-response function, and hence strategies are ex post optimal.

Figure 2: *Demand functions for a local market with three households.*



(a) $\theta_i = 4, \varepsilon_i = 1, q_i = 0$ (b) $\theta_i = 4, \varepsilon_i = 1, q_i = 1$

to standard demand reduction equilibria (e.g., Ausubel et al., 2014), where bids for the first unit are equal to marginal utility and demand is more understated for each additional unit. As apparent in panel (b), also the household with $q_i = 1$ strategically reduces demand. However, as compared to the demand curve of its neighbors with no solar output, this household shifts demand upward when selling. This household marks-up its demand, as it intends to increase the market price to its favor. For all $X_i(p, \varepsilon_i) > q_i = 1$ this household demands electricity at prices lower than marginal utility, while for $X_i(p, \varepsilon_i) < q_i = 1$ it is willing to sell electricity at a mark-up on marginal utility.

Using Lemma 1 and equation (8), the equilibrium demand schedule becomes

$$X_i^*(p, \varepsilon_i) = \frac{n-2}{n-1}(\theta_i + \varepsilon_i - p) + \frac{1}{n-1} q_i, \tag{12}$$

and the market clearing condition in (2) yields

$$\sum_i^n \left(\frac{n-2}{n-1}(\theta_i + \varepsilon_i - p) + \frac{1}{n-1} q_i \right) = \sum_i^n q_i. \tag{13}$$

The equilibrium price is

$$p^* = \frac{1}{n}\sum_{i}^{n}(\theta_i + \varepsilon_i - q_i). \tag{14}$$

and depends only on market fundamentals. We summarize the above findings in the following two corollaries.

Corollary 1. *The equilibrium market price is independent of demand reduction strategies and only depends on market fundamentals for demand, θ_i and ε_i, and supply q_i of all households $i = 1, .., n$. Demand functions determine market shares in consumption at the competitive market price.*

Corollary 2. *Strategically announced demand in equilibrium leads to allocative inefficiency. Households that are net-buyers from the market consume too little. Households that are net-sellers to the market consume too much energy.*

First, Corollary 1 illustrates that the equilibrium price is independent of bid shading, because strategies of net-buying and net-selling households cancel out. To see this, consider the case where $n-1$ households have zero supply and only one household i generates electricity. In equilibrium, the market supply from household i of $q_i - X_i(p, \varepsilon_i)$ must equal demand of the $n-1$ buying households, $\sum_{j \neq i}^{n} X_j(p, \varepsilon_j)$. The demand reduction of the buying households will be exactly offset by the selling household. This requires that the selling household reduces its supply by announcing higher demand and consuming more electricity. The household consumes more electricity, because the cost of consumption declines when at the same time the price of its supply increases. Furthermore, note that the equilibrium market price only depends on the mean household size, and that asymmetric households, i.e., the variance in household size, does not impact the equilibrium price.

Second, Corollary 2 follows from re-visiting Figure 2 that shows that at any potentially realized equilibrium price, announced demand is below true demand for net-buying households, while announced demand is above true demand for net-selling households. Furthermore, Corollary 2 follows from comparing equilibrium demand schedules in (12) to the true demand in equation (8).[13]

Finally, notice that although households can always buy energy at the competitive market price, investment still matters, because the distribution of generation assets among households impacts their consumption shares.

III(i). Investment

We consider investment to take place prior to market clearing and prior to announcing demand. At the investment stage, households consequently do not know their realized demand shock ε_i. In addition, households have to form a prior on the demand shock of their neighbors.

To separate out the different demand shocks, define $\sum_{j \neq i}^{n} \varepsilon_j := \Psi_i$ with $g_i(\Psi_i)$ being the density function of Ψ_i. Recalling that $\varepsilon_i \in [-\varepsilon_o, \varepsilon_o]$, Ψ_i must be distributed in $[-(n-1)\varepsilon_o, (n-1)\varepsilon_o]$. Using this definition, the equilibrium price in (14) can be rewritten as $p^* = \frac{1}{n}\left[\Psi_i + \varepsilon_i + \sum_i^n (\theta_i - q_i)\right]$. Household i finds its optimal investment by maximizing expected utility in (5) net of investment costs, weighted over all possible demand shocks:

$$(15)\quad \mathbb{E}[EU_i] = \int_{-\varepsilon_o}^{\varepsilon_o} \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \left[U_i(X_i(p^*, \varepsilon_i), \varepsilon_i) + p^*(q_i - X_i(p^*, \varepsilon_i))\right] g_i(\Psi_i) d\Psi_i f(\varepsilon_i) d\varepsilon_i - p_s q_i$$

where p_s is the market price for solar units and represents the investment cost per solar unit. The first order condition for household i's optimal investment choice becomes[14]

$$(16)\qquad q_i = \theta_i - (n-1)p_s - \frac{n-2}{n} \sum_i^n (q_i - \theta_i).$$

[13] Note that larger households reduce demand by more and thus cause relatively larger inefficiencies.
[14] We focus on interior solutions.

Equation (16) shows that individual investment in solar plants is a strategic substitute to aggregate market investment. Summing up the optimality condition over all n households, we obtain $\sum_i^n q_i = \sum_i^n \theta_i - n\left((n-1)p_s - \frac{n-2}{n}\sum_i^n (q_i - \theta_i)\right)$. Solving this expression for $\sum_i^n q_i$ yields aggregate equilibrium investment of

$$\sum_i^n q_i^* = \sum_i^n \theta_i - np_s. \tag{17}$$

Last, when substituting the aggregate equilibrium investment in (17) into the expression for optimal investment of household i in (16), rearranging yields $q_i^* = \theta_i - p_s$. This solution is straightforward: each household only invests in solar plants as long as its maximum valuation for electricity, θ_i, is above the price of solar plants. We summarize this finding in the next Proposition.

Proposition 3. *Household i's equilibrium investment in generation assets is $q_i = \theta_i - p_s$.*

Proof. The result follows from equation (17) and is derived in full in Appendix B.I.. □

It follows that larger households contribute with relatively more supply to the local market. Using Proposition 3 and equation (14), the equilibrium power price—given optimal investment—eventually becomes $p^* = \frac{1}{n}\sum_i^n(\varepsilon_i + p^s)$. With $E[\varepsilon_i] = 0$, the expected power price of the local market simply is

$$\mathbb{E}[p^*] = p_s, \tag{18}$$

implying that, for optimal investment levels, the expected electricity price in local energy markets equals the costs of generation assets.

IV. A clearing mechanism for truthful bidding

Our results so far show that energy prices are competitive and reflect the costs of generating units, but that strategic household behavior introduces allocative inefficiency. The results of this section illustrate that a microgrid operator can impose additional payments and network tariffs to induce truth-telling behavior, mitigate allocative inefficiency and guarantee first-best welfare. To see this, consider the expected utility of household i when having to pay $Z(X_i(p,\varepsilon_i),q_i)$ to the microgrid operator:

$$EU_i = \int_{\underline{p}}^{\overline{p}} \left[U_i\left(X_i(p,\varepsilon_i),\varepsilon_i\right) - Z(X_i(p,\varepsilon_i),q_i) + p(q_i - X_i(p,\varepsilon_i))\right] dH_i(p, X_i(p,\varepsilon_i)). \tag{19}$$

As can be seen in equation (19), we consider a clearing mechanism where households pay the uniform energy price, p, plus an additional charge Z. Maximizing and proceeding as above, the first order condition can be written as

$$\frac{\partial}{\partial X_i} U_i\left(X_i(p,\varepsilon_i),\varepsilon_i\right) = p + \frac{\partial}{\partial X_i} Z(X_i(p,\varepsilon_i),q_i) + (q_i - X_i(p,\varepsilon_i))\,\frac{H_{X_i}(p,X_i(p,\varepsilon_i))}{H_p(p,X_i(p,\varepsilon_i))}. \tag{20}$$

Note that if the last two terms on the right hand side cancel out, it is optimal for household i to announce true demand. Therefore, we obtain the following Proposition.

Proposition 4. *With a price p determined by market clearing and additional payments*

$$Z(X_i(p,\varepsilon_i),q_i) = \int_{q_i}^{X_i(p,\varepsilon_i)} \frac{q_i - x}{n-1} dx + \frac{\sigma}{2n}$$

imposed by the microgrid operator, there exists an equilibrium such that households submit their true demand $X_i(p) = \theta_i + \varepsilon_i - p$.

Proof. Assuming truthful bidding of all other households $j \neq i$ and using

$$\frac{H_{X_i}(p, X_i(p, \varepsilon_i))}{H_p(p, X_i(p, \varepsilon_i))} = -\frac{1}{n-1} \text{ as well as } \frac{\partial}{\partial X_i} Z(X_i(p, \varepsilon_i), q_i) = \frac{q_i - X_i}{n-1}$$

the first order condition in (20) reduces to

$$\frac{\partial U_i(X_i(p, \varepsilon_i), \varepsilon_i)}{\partial X_i} = p \text{ and thus } X_i(p, \varepsilon_i) = \theta_i + \varepsilon_i - p,$$

so that equilibrium demand functions are truth-telling. □

The main point is that with truthful bidding of all other households the marginal payment

(21) $$\frac{\partial}{\partial X_i} Z(X_i(p, \varepsilon_i), q_i) = \frac{q_i - X_i}{n-1}$$

exactly offsets the strategic incentives for household i to deviate from truthful bidding. Note further, that according to the first term in $Z(X_i(p, \varepsilon_i), q_i)$, net sellers, i.e., $q_i > X_i(p, \varepsilon_i)$, as well as net buyers, i.e., $q_i < X_i(p, \varepsilon_i)$, receive a subsidy from the microgrid operator:

(22) $$\int_{q_i}^{X_i(p, \varepsilon_i)} \frac{q_i - x}{n-1} dx < 0 \text{ for both } q_i > X_i(p, \varepsilon_i) \text{ and } q_i < X_i(p, \varepsilon_i).$$

Notice that the above subsidy mechanism draws from the idea in Loeb and Magat (1979) on optimal utility regulation. More specifically, the microgrid operator's payments to the households stated in equation (22) are similar to the payments that regulators pay to utilities for the consumer surplus they generate. The one-off network charge paid by participating households in Proposition 4 of $\frac{\sigma}{2n}$ corresponds to the payments generated in the franchise bidding stage of the Loeb-Magat mechanism. Importantly, to implement the above mechanism it suffices if the microgrid operator knows the structure of household utility and observes the amount of installed generation assets q_i. As in Loeb and Magat (1979), the mechanism requires information on the slope of demand. Yet, the microgrid operator does not require any data on the household's private information, i.e., on their demand shock ε_i.

IV(i). Welfare and microgrid operator's profit

We conclude our analysis by investigating the welfare implications of our proposed mechanism, including profits of the microgrid operator. First, notice that the network payments Z depend on household i's investment decision when facing network tariffs. We therefore start by showing that the above introduced clearing mechanism does not change equilibrium investment. Solving the market clearing condition

$$\sum_{i}^{n} (\alpha_i + \beta_i \varepsilon_i - \gamma_i p) = \sum_{i}^{n} q_i \tag{23}$$

for true demand with $\alpha_i = \theta_i$ and $\beta = \gamma = 1$ yields the identical market price as stated in equation (2) for the equilibrium with strategic households. In a similar fashion, re-visiting expected utility at the investment stage in equation (15) and using truthful demand yields equilibrium investment of

$$q_i = \theta_i - p^s. \tag{24}$$

As can be seen, equilibrium investment again is identical to the case with strategic demand in equation (17) above. This finding allows to state the next Proposition on welfare and the budget of the microgrid operator.

Proposition 5. *With a price p determined by market clearing and additional payments*

$$Z(X_i(p, \varepsilon_i), q_i) = \int_{q_i}^{X_i(p,\varepsilon_i)} \frac{q_i - x}{n-1} dx + \frac{\sigma}{2n}$$

imposed by the microgrid operator, there exists an equilibrium such that

(1) households invest according to $q_i = \theta_i - p_s$

(2) the expected profit of the network operator is equal to zero

(3) all households are better off as compared to the market without such payments.

Proof. The derivation of equilibrium investment can be found in Appendix B.II.. Using $q_i = \theta_i - p^s$ and taking expectations over $Z(X_i(p, \varepsilon_i), q_i)$, the expected revenue of the microgrid operator turns out to be zero $\mathbb{E}[\Pi] = 0$ which proves the second statement above. To prove the third statement, we use equilibrium prices, investment, and demand for the truth-telling equilibrium and find utility of

$$\mathbb{E}[EU_i^*] = \frac{1}{2}\left((\theta_i - p_s)^2 + \frac{n-1}{n}\sigma\right),$$

where σ denotes the variance of the demand shock, $\mathbb{E}[\varepsilon_i^2]$. Using strategic demand instead, we find utility of

$$\mathbb{E}[EU_i] = \frac{1}{2}\left((\theta_i - p_s)^2 + \frac{n-2}{n-1}\sigma\right)$$

with

$$\mathbb{E}[EU_i^*] - \mathbb{E}[EU_i] = \frac{\sigma}{2(n-1)n} > 0,$$

so that households are better off as compared to the market without additional payments. □

As shown, welfare is strictly higher under the truth-telling mechanism with network tariffs as compared to the strategic equilibrium. Notice that the difference in welfare is independent of the distribution of household sizes. Because market prices and investment are identical for the two cases, this result is entirely driven through the increase in allocative efficiency from truth-telling behavior.[15] Further, notice that the expected profit of the microgrid operator

[15]Full welfare expressions can be computed using a proxy for the distribution of the demand shock. For instance, with ε_i being uniformly distributed and Ψ_i following an Irwin-Hall distribution, expected utility for the truth-telling equilibrium equals $\frac{1}{2}\left(\theta_i - p_s\right)^2 + \frac{n-1}{6n}\varepsilon_0^2$.

is zero as a result of the fixed one-off payment $\frac{\sigma}{2n}$, which has however no strategic effect on household behavior. Therefore, the mechanism above increases welfare and is budget balanced for the microgrid operator.

As a final corollary result, Proposition 5 shows that the difference in welfare of $\frac{\sigma}{2(n-1)n}$ depends on the number of households connected to the microgrid and decreases as the market grows. This is because strategic demand and allocative inefficiency vanish for a larger number of households. Hence the proposed clearing mechanism increases welfare especially in small markets. At the same time, this finding points to the benefits of interoperability for local energy markets and shows that investing in converters or rectifiers to connect AC and DC standards always increases welfare, if the costs of connecting markets are sufficiently low. As a consequence, where small markets cannot be integrated into a larger power system, market design and clearing rules become essential to the efficient provision of locally generated energy.

V. Conclusion

The provision of electricity is increasingly organized in local energy markets. In this article, we provide a model to study pricing behavior, investment incentives, and welfare in local energy markets.

We have derived a set of positive efficiency results. First, local energy markets can provide electricity at competitive prices. Importantly, this result holds also for a small number of participating households and despite strategic demand reduction. Furthermore, combining generation and trading possibilities in local markets yields efficient investment incentives so that market prices reflect the costs of investing in generating units. However, we show that these positive efficiency results can deteriorate due to allocative inefficiency. This is, households with large generation capacity that are selling energy to the market typically consume too much and "share" too little energy with the market. This effect results from their efforts to increase market prices. In turn, households that are net-buyers will typically consume too little energy as compared to their optimal consumption levels. Yet, allocative inefficiency vanishes and welfare increases the larger markets become. This finding highlights the value of interoperability so that connecting adjacent markets can increase overall welfare, if costs of connecting markets are sufficiently low.

To mitigate strategic behavior, we have proposed a clearing mechanism that includes energy payments and network tariffs. We have shown that this mechanism can avoid strategic demand reduction and restore first-best welfare in local energy markets. We believe this mechanism is applicable, as it merely requires knowledge by the microgrid operator on the number of participating households, their size and basic preferences, but does not require the microgrid operator to know the exact demand realization of each household at any given point in time.

In sum, the results of this article point to the efficiency of local energy markets, and underscore the importance of market design and payment mechanisms to make households benefit from the shift towards generation and trading in local energy markets.

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Appendix

A Equilibrium demand functions

A.I. Derivation of the optimality condition

Integrating (5) by parts yields

$$\begin{aligned}
EU_i &= \int_{\underline{p}}^{\overline{p}} \left[U_i\left(X_i(p,\varepsilon_i),\varepsilon_i\right) + p(q_i - X_i(p,\varepsilon_i))\right] dH_i(p, X_i(p,\varepsilon_i)) \\
&= \left[U_i\left(X_i(p,\varepsilon_i),\varepsilon_i\right) + p(q_i - X_i(p,\varepsilon_i))\right] H_i(p, X_i(p,\varepsilon_i))\Big|_{\underline{p}}^{\overline{p}} \\
&- \int_{\underline{p}}^{\overline{p}} \frac{d}{dp}\left[U_i\left(X_i(p,\varepsilon_i),\varepsilon_i\right) + p(q_i - X_i(p,\varepsilon_i))\right] H_i(p, X_i(p,\varepsilon_i))dp.
\end{aligned}$$

Because $H_i(\underline{p}) = 0$ and $H_i(\bar{p}) = 1$ we obtain

$$\begin{aligned}
EU_i &= U_i\left(X_i(\overline{p},\varepsilon_i),\varepsilon_i\right) + \overline{p}(q_i - X_i(\overline{p},\varepsilon_i)) \\
&- \int_{\underline{p}}^{\overline{p}} \left[\left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) X_i'(p,\varepsilon_i) + q_i - X_i(p,\varepsilon_i)\right] H_i(p, X_i(p,\varepsilon_i))dp.
\end{aligned}$$

Therefore households maximize

$$\max_{X_i(p,\varepsilon_i)} \left[U_i\left(X_i(\overline{p},\varepsilon_i),\varepsilon_i\right) + \overline{p}(q_i - X_i(\overline{p},\varepsilon_i)) - \int_{\underline{p}}^{\overline{p}} \mathcal{L}(p, X_i(p,\varepsilon_i), X_i'(p))dp\right]$$

with

$$\mathcal{L}(p, X(p), X_i'(p)) := \left[\left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) X_i'(p) + q_i - X_i(p,\varepsilon_i)\right] H_i(p, X_i(p,\varepsilon_i)).$$

With free salvage value (Kamien and Schwartz, 2012), the first order condition for an unspecified $X_i(\overline{p}, \varepsilon_i)$ becomes

$$\frac{d}{dp}\mathcal{L}_{X'} = \mathcal{L}_X$$

with

$$\left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) - \mathcal{L}_{X'} = 0 \text{ for } p = \overline{p}.$$

Computing the derivatives yields

$$\mathcal{L}_{X'} = \left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) H_i(p, X_i(p,\varepsilon_i))$$

$$\begin{aligned}\mathcal{L}_X = H_{X_i} &\left[\left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) X_i'(p,\varepsilon_i) + q_i - X_i(p,\varepsilon_i)\right] \\ &+ \left[\frac{\partial^2 U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i^2} X_i'(p,\varepsilon_i) - 1\right] H_i(p, X_i(p,\varepsilon_i))\end{aligned}$$

and

$$\begin{aligned}\frac{d}{dp}\mathcal{L}_{X'} = &(H_p(p, X_i(p,\varepsilon_i)) + H_{X_i}(p, X_i(p,\varepsilon_i))X_i'(p,\varepsilon_i)) \left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) \\ &+ \left(\frac{\partial^2 U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i^2} X_i'(p) - 1\right) H_i(p, X_i(p,\varepsilon_i)).\end{aligned}$$

Using the above and rearranging $\frac{d}{dp}\mathcal{L}_{X'} = \mathcal{L}_X$ yields equation (6) in the main text

$$\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p = (q_i - X_i(p,\varepsilon_i)) \frac{H_{X_i}(p, X_i(p,\varepsilon_i))}{H_p(p, X_i(p,\varepsilon_i))}.$$

Last, for $p = \overline{p}$ it must hold that

$$\left(\frac{\partial U_i(X_i(\overline{p},\varepsilon_i),\varepsilon_i)}{\partial X_i} - \overline{p}\right) - \mathcal{L}_{X'} = \left(\frac{\partial U_i(X_i(\overline{p},\varepsilon_i),\varepsilon_i)}{\partial X_i} - \overline{p}\right) - \left(\frac{\partial U_i(X_i(\overline{p},\varepsilon_i),\varepsilon_i)}{\partial X_i} - \overline{p}\right) = 0.$$

A.II. Solving for linear strategies

First, to obtain the derivatives of the price distribution H_{X_i} and H_p, use the demand function $X_i(p, \varepsilon_i) = \alpha_i + \beta_i\varepsilon_i - \gamma_i p$ and rearrange equation (4) in the main text as

$$\begin{aligned}
H_i(p, X_i(p)) &= Pr\left(p^* \leq p \mid X_i\right) \\
&= Pr\left(\sum_{j\neq i}^{n} X_j(p, \varepsilon_j) + X_i \leq \sum_{i}^{n} q_i \;\middle|\; X_i\right) \\
&= Pr\left(\sum_{j\neq i}^{n} (\alpha_j + \beta_j\varepsilon_j - \gamma_j p) + X_i \leq \sum_{i}^{n} q_i \;\middle|\; X_i\right) \\
&= Pr\left(\sum_{j\neq i}^{n} \beta_j\varepsilon_j \leq \sum_{i}^{n} q_i - \sum_{j\neq i}^{n} (\alpha_j - \gamma_j p) - X_i \;\middle|\; X_i\right).
\end{aligned}$$

Let $\mathcal{F}$ be the distribution function of $\sum_{j\neq i}^{n} \beta_j\varepsilon_j$ with density $\mathcal{F}'$. Differentiating with respect to X_i yields

$$\begin{aligned}
\frac{\partial}{\partial X_i}\mathcal{F}\left(\sum_{j\neq i}^{n} \beta_j\varepsilon_j \leq \sum_{i}^{n} q_i - \sum_{j\neq i}^{n} (\alpha_j - \gamma_j p) - X_i\right) &= \mathcal{F}'(\cdot)\frac{\partial}{\partial X_i}\left(\sum_{i}^{n} q_i - \sum_{j\neq i}^{n} (\alpha_j - \gamma_j p) - X_i\right) \\
&= -\mathcal{F}'(\cdot)
\end{aligned}$$

and differentiating with respect to p gives

$$\begin{aligned}
\frac{\partial}{\partial p}\mathcal{F}\left(\sum_{j\neq i}^{n} \beta_j\varepsilon_j \leq \sum_{i}^{n} q_i - \sum_{j\neq i}^{n} (\alpha_j - \gamma_j p) - X_i\right) &= \mathcal{F}'(\cdot)\frac{\partial}{\partial p}\left(\sum_{i}^{n} q_i - \sum_{j\neq i}^{n} (\alpha_j - \gamma_j p) - X_i\right) \\
&= \mathcal{F}'(\cdot)\sum_{j\neq i}^{n} \gamma_j.
\end{aligned}$$

Finally, we have

$$\frac{H_{X_i}}{H_p} = \frac{-1}{\sum_{j\neq i}^{n} \gamma_j}.$$

When using the above for the optimality condition in equation (6), we obtain equation (9) in the main text:

$$\theta_i + \varepsilon_i - p - (\alpha_i + \beta_i\varepsilon_i - \gamma_i p) + \frac{1}{\sum_{j\neq i}^{n} \gamma_j}(q_i - (\alpha_i + \beta_i\varepsilon_i - \gamma_i p)) = 0.$$

Applying coefficient matching to solve the first order condition yields

$$0 = \varepsilon_i - \varepsilon_i\beta_i - \frac{\varepsilon_i\beta_i}{\sum_{j\neq i}^n \gamma_j}$$

$$0 = -p + p\gamma_i + \frac{p\gamma_i}{\sum_{j\neq i}^n \gamma_j}$$

$$0 = -\alpha_i + \theta_i + \frac{q_i - \alpha_i}{\sum_{j\neq i}^n \gamma_j}.$$

with solutions as in Lemma 1. The solution for $n < 3$ is trivial with $\alpha_i = q_i$ and $\beta_i = \gamma_i = 0$. To see this solution, multiply both sides of the optimality condition by $\sum_{j\neq i}^n \gamma_j$ and plug in $(\alpha_i, \beta_i, \gamma_i) = (q_i, 0, 0)$. For $n < 3$, this is the only equilibrium candidate that survives. In this equilibrium, all trade breaks down and each household consumes its own production. In this case the stand-alone utility of household i is straightforward to calculate as

$$EU_i = \int_{-\varepsilon_o}^{\varepsilon_o} \left[(\theta_i + \varepsilon_i)q_i - \frac{1}{2}q_i^2\right] f(\varepsilon_i)d\varepsilon_i - p_s q_i = \theta_i q_i - \frac{1}{2}q_i^2 - p_s q_i$$

which is maximized at $q_i^* = \theta_i - p_s$, providing utility of $\frac{1}{2}(\theta_i - p_s)^2$.

Last, we show that only symmetric strategies exist for all positive γ_i. We proceed by induction arguments. Consider two households i and j, plus the remaining $n-2$ households with (ex-ante possibly distinct) γ_k. From the derivation of bidding strategies we know that $\gamma_i = \frac{\sum_{j\neq i}^n \gamma_j}{1+\sum_{j\neq i}^n \gamma_j}$. Hence, we can write the optimality conditions for households i and j in equation (11) as

$$0 = \gamma_i - \frac{\gamma_j + \sum_{k\neq i,j}^n \gamma_k}{1 + \gamma_j + \sum_{k\neq i,j}^n \gamma_k}$$

$$0 = \gamma_j - \frac{\gamma_i + \sum_{k\neq i,j}^n \gamma_k}{1 + \gamma_i + \sum_{k\neq i,j}^n \gamma_k}$$

and solve for γ_i and γ_j. The only solution with positive γ_i is $\gamma_i = \gamma_j$. Hence $s = 2$ households (i and j) exist with symmetric strategies $\gamma_i = \gamma_j$. Now taking one additional household l out of $\sum_k^n \gamma_k$ with $k \neq i, j$ we can write

$$0 = \gamma_i - \frac{(s-1)\gamma_i + \gamma_l + \sum_{k\neq i,j,l}^n \gamma_k}{1 + (s-1)\gamma_i + \gamma_l + \sum_{k\neq i,j,l}^n \gamma_k}$$

$$0 = \gamma_l - \frac{s\gamma_i + \sum_{k\neq i,j,l}^n \gamma_k}{1 + s\gamma_i + \sum_{k\neq i,j,l}^n \gamma_k}$$

with $s = 2$. We again solve for γ_i and γ_l and again find only one solution with positive parameters, which is the symmetric solution. Continuing with $\gamma_i = \gamma_j = \gamma_l$ and $s = 3$ yields one additional symmetric household. Continue until $s = n - 1$ for which all households' γ_i are symmetric.

A.III. Existence of linear equilibria

The first order condition for household i can be written as

$$\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p = (q_i - X_i(p,\varepsilon_i))\,\frac{-1}{\sum_{j\neq i}^{n}\gamma_j}.$$

Substituting in the utility function, writing $\sum_{j\neq i}^{n}\gamma_j =: \Gamma_{-i}$, and rearranging yields

$$X_i(p) = \frac{q_i + \Gamma_{-i}(\varepsilon - p + \theta_i)}{1+\Gamma_{-i}}.$$

Note that the above condition must hold for any price level p. The corresponding derivative with respect to p,

$$X_i'(p) = -\frac{\Gamma_{-i}}{1+\Gamma_{-i}},$$

is linear. Therefore, the best reply of household i to linear strategies of all other households $j \neq i$ is to likewise play a linear strategy.

For the proof of sufficiency, use again the functional from Appendix A.I.:

$$\int_{\underline{p}}^{\overline{p}} \mathcal{L}(p, X_i(p), X_i'(p))dp = \int_{\underline{p}}^{\overline{p}} \left[\left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) X_i'(p) + q_i - X_i(p,\varepsilon_i)\right] H_i(p, X_i(p,\varepsilon_i))dp.$$

Considering any $\eta(p)$, the second order Taylor expansion of $\int_{\underline{p}}^{\overline{p}} \mathcal{L}(p, X_i(p) + \alpha\eta(p), X_i'(p) + \alpha\eta'(p))dp$ with respect to α can be written as (see Liberzon (2011) section 2.6.1):

$$\begin{aligned}
&\int_{\underline{p}}^{\overline{p}} \left[\mathcal{P}(p)\eta(p)^2 + \mathcal{Q}(p)(\eta(p))^2\right] dp \\
&\text{with } \mathcal{P}(p) := \frac{1}{2}\mathcal{L}_{X_i'(p)X_i'(p)}(p, X_i(p), X_i'(p)) \\
&\text{and } \mathcal{Q}(p) := \frac{1}{2}\left(\mathcal{L}_{X_i(p)X_i(p)} - \frac{d}{dp}\mathcal{L}_{X_i(p)X_i'(p)}\right)
\end{aligned}$$

Because $\mathcal{L}(p, X_i(p), X_i'(p))$ is linear in $X_i'(p)$ we have $\mathcal{P}(p) = 0$. Furthermore, $\mathcal{Q}(p)$ can be rewritten to

$$-H_{X_i} - H_p U_{X_i X_i}(X_i(p), \varepsilon_i) + (q - X_i(p)) H_{X_i X_i} + (p - U_{X_i}(X_i(p), \epsilon)) H_{p X_i}.$$

Next, recalling from Appendix A.I. that $H_{X_i} = -\mathcal{F}'(\cdot)$ and $H_p = \mathcal{F}'(\cdot) \sum_{j \neq i}^{n} \gamma_j$, and because utility is quadratic in $X_i(p)$ so that $U_{X_i X_i}(X_i(p), \varepsilon_i) = -1$, the first two terms simplify to

$$\mathcal{F}'(\cdot) + \mathcal{F}'(\cdot) \sum_{j \neq i}^{n} \gamma_j > 0.$$

Similarly, the remaining two terms can be rewritten to

$$\frac{\partial \mathcal{F}'(\cdot)}{\partial X_i} \left((q - X_i(p)) - (p - U_{X_i}(X_i(p), \epsilon)) \sum_{j \neq i}^{n} \gamma_j \right) = 0,$$

which follows when substituting $U_{X_i}(X_i(p), \epsilon)) = \theta_i + \epsilon_i - X_i(p)$ and using the equilibrium demand curve $X_i(p)$ from equation (12). Hence, the second variation is positive and the second order conditions are satisfied, implying a maximum in utility. To see this, recall that households solve

$$\max_{X_i(p)} \left[U_i \left(X_i(\overline{p}), \varepsilon_i \right) + \overline{p}(q_i - X_i(\overline{p})) - \int_{\underline{p}}^{\overline{p}} \mathcal{L}(p, X_i(p), X_i'(p)) dp \right].$$

B Equilibrium investment

B.I. Investment with strategic demand

Starting from expected utility

$$\mathbb{E}[EU_i] = \int_{-\varepsilon_o}^{\varepsilon_o} \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \left[U_i(X_i(p^*), \varepsilon_i) + p^*(q_i - X_i(p^*)) \right] g_i(\Psi_i) d\Psi_i f(\varepsilon_i) d\varepsilon_i - p_s q_i$$

we take the first order condition and arrive at

$$\int_{-\varepsilon_o}^{\varepsilon_o} \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \left[\frac{2(n-1)(\theta_i + \varepsilon_i - q_i) + (n-2)\left(\sum_{j \neq i}^{n} \theta_j + \Psi_i - \sum_{j \neq i}^{n} q_j\right)}{n(n-1)} \right] g_i(\Psi_i) d\Psi_i f(\varepsilon_i) d\varepsilon_i = p_s$$

We then solve the inner integral by parts using that $G(-(n-1)\varepsilon_o) = 0$ and $G((n-1)\varepsilon_o) = 1$. Further, we use that for a symmetric PDF g_i with a mean of zero, the anti-derivative of its CDF, $\overline{G}(\Psi_i) = \int G(\Psi_i) d\Psi_i$, evaluated at the bound of the support yields $\overline{G}((n-1)\varepsilon_o) = \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} G(\Psi_i) d\Psi_i = \Psi_i G(\Psi_i)|_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} - \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \Psi_i g(\Psi_i) d\Psi_i = (n-1)\varepsilon_o$ and $\overline{G}(-(n-1)\varepsilon_o) = 0$. Using a corresponding procedure for the outer integral yields equation (16) in the main text.

B.II. Investment with truthful demand

Substituting the true demand curve $X_i(p, \varepsilon_i) = \theta_i + \varepsilon_i - p$ into expected utility in equation (15), taking expectations, and differentiating with respect to investment leads to

$$\frac{\partial \mathbb{E}[EU_i]}{\partial q_i} = \frac{1}{n} \left[\sum_{j \neq i}^{n} q_j + q_i - \sum_{j \neq i}^{n} \theta_j - \theta_i \right] - p_s$$

which also implies

$$\sum_{j \neq i}^{n} q_j = \sum_{j \neq i}^{n} \theta_j - (n-1)p_s \Longleftrightarrow q_i = \theta_i - p_s.$$